

# Contents

<b>Braids and Knots</b> .....	1
Patrick D. Bangert	
1 Physical Knots and Braids: A History and Overview .....	2
2 Braids and the Braid Group .....	4
2.1 The Topological Idea .....	4
2.2 The Origin of Braid Theory .....	5
2.3 The Topological Braid .....	9
2.4 The Braid Group .....	12
2.5 Other Presentations of the Braid Group .....	16
2.6 The Alexander and Jones Polynomials .....	18
2.7 Properties of the Braid Group .....	21
2.8 Algorithmic Problems in the Braid Groups .....	22
3 Braids and Knots .....	24
3.1 Notation for Knots .....	24
3.2 Braids to Knots .....	28
3.3 Example: The Torus Knots .....	28
3.4 Knots to Braids I: The Vogel Method .....	29
3.5 Knots to Braids II: An Axis for the Universal Polyhedron ....	31
3.6 Peripheral Group Systems of Closed Braids .....	39
4 Classification of Braids and Knots .....	45
4.1 The Word Problem I: Garside's Solution .....	45
4.2 The Word Problem II: Rewriting Systems .....	47
4.3 The Conjugacy Problem I: Garside's Solution .....	53
4.4 The Conjugacy Problem II: Rewriting Systems .....	54
4.5 Markov's Theorem .....	59
4.6 The Minimal Word Problem .....	62
5 Open Problems .....	69
References .....	70

<b>Topological Quantities: Calculating Winding, Writhing, Linking, and Higher Order Invariants</b> . . . . .	75
Mitchell A. Berger (CIME Lecturer)	
1 Introduction . . . . .	75
2 Winding Numbers . . . . .	77
2.1 Two Braided Curves between Parallel Planes . . . . .	77
2.2 General Curves . . . . .	78
2.3 Topological Invariance . . . . .	81
3 Linking Numbers . . . . .	82
3.1 Winding Number Derivation . . . . .	82
3.2 General Properties . . . . .	83
4 Twist and Writhe Numbers . . . . .	84
4.1 Ribbons . . . . .	84
4.2 Twisted Tubes . . . . .	87
5 Writhe from Winding Numbers . . . . .	87
5.1 The Twist as a Function of Height . . . . .	89
5.2 The Local Winding Number as a Function of Height . . . . .	90
5.3 The Local Writhe as a Function of Height . . . . .	91
5.4 The Nonlocal Winding Number as a Function of Height . . . . .	91
5.5 Example: A Trefoil Torus Knot . . . . .	92
6 Writhe for Open Curves . . . . .	94
7 Higher Order Winding . . . . .	96
References . . . . .	97
<b>Tangles, Rational Knots and DNA</b> . . . . .	99
Louis H. Kauffman (CIME Lecturer) and Sofia Lambropoulou	
1 Introduction . . . . .	99
2 2-Tangles and Rational Tangles . . . . .	102
3 Continued Fractions and the Classification of Rational Tangles . . . . .	107
4 Alternate Definitions of the Tangle Fraction . . . . .	111
4.1 $F(T)$ Through the Bracket Polynomial . . . . .	112
4.2 The Fraction through Coloring . . . . .	120
4.3 The Fraction through Conductance . . . . .	122
5 The Classification of Unoriented Rational Knots . . . . .	123
6 Rational Knots and Their Mirror Images . . . . .	127
7 The Oriented Case . . . . .	128
8 Strongly Invertible Links . . . . .	132
9 Applications to the Topology of DNA . . . . .	133
References . . . . .	138
<b>The Group and Hamiltonian Descriptions of Hydrodynamical Systems</b> . . . . .	141
Boris Khesin (CIME Lecturer)	
1 Introduction . . . . .	141
2 Euler Equations and Geodesics . . . . .	142
2.1 The Euler Equation of an Ideal Incompressible Fluid . . . . .	142

2.2	Geodesics on Lie Groups	143
2.3	Geodesic Description for Various Equations	144
3	Euler Equations on Groups as Hamiltonian Systems and the Binormal Equation	144
3.1	Hamiltonian Reformulation of the Euler Equations	144
3.2	Hamiltonian Structure of the Landau-Lifschitz Equation	145
3.3	Properties of the Binormal Equation	147
4	The KdV-Type Equations as Euler Equations	149
4.1	The Virasoro Algebra and the KdV Equation	149
4.2	Similar Equations and Conservation Laws	151
5	Hamiltonian Structure of the Euler Equations for an Incompressible Fluid	152
5.1	The Euler Hydrodynamics as a Hamiltonian Equation	152
5.2	The Space of Knots and the Dual of the Lie Algebra of Divergence-Free Vector Fields	155
	References	156
	<b>Singularities in Fluid Dynamics and their Resolution</b>	159
	H.K. Moffatt (CIME Lecturer)	
1	Introduction	159
2	Boundary-Driven Singularities	160
3	Cusp Singularities at a Free Surface	162
4	A Simple Finite-Time Singularity: the Euler Disk	163
5	Finite-Time Singularities at Interior Points	165
	References	168
	<b>Structural Complexity and Dynamical Systems</b>	169
	Renzo L. Ricca (School Director and CIME Lecturer)	
1	Introduction	169
2	Helmholtz's Work on Vortex Motion: Birth of Topological Fluid Mechanics	170
2.1	Multi-Valued Potentials in Multiply Connected Regions	170
2.2	Green's Theorem in Multiply Connected Regions	174
2.3	Conservation Laws	175
3	Measures of Structural Complexity	175
3.1	Dynamical Systems and Vector Field Analysis	176
3.2	Measures of Tangle Complexity	177
4	Topological Bounds on Energy and Helicity-Crossing Number Relations for Magnetic Knots and Links	184
4.1	Topology Bounds Energy in Ideal Fluid	185
4.2	Helicity-Crossing Number Relations in Dissipative Fluid	187
	References	187

<b>Random Knotting: Theorems, Simulations and Applications ..</b>	<b>189</b>
De Witt Sumners (CIME Lecturer)	
1 Introduction .....	189
2 The Frisch-Wasserman-Delbruck Conjecture .....	191
3 Entanglement Complexity of Random Knots and Random Arcs .....	196
4 Writhe, Signature and Chirality of Random Knots .....	198
5 Application of Random Knotting to Viral DNA Packing .....	203
5.1 Knot Type Probabilities for P4 DNA in Free Solution .....	206
5.2 Monte Carlo Simulation .....	207
5.3 Results and Discussion Knot Complexity of DNA Molecules Extracted from Phage P4 .....	208
5.4 Identification of Specific Knot Types by Their Location on the Gel .....	211
5.5 Monte Carlo Simulations of Random Knot Distributions in Confined Volumes .....	211
References .....	215
<b>Index .....</b>	<b>221</b>



<http://www.springer.com/978-3-642-00332-5>

Random Polymers

École d'Été de Probabilités de Saint-Flour XXXVII - 2007

Hollander, F.

2009, XIII, 258 p. 84 illus., Softcover

ISBN: 978-3-642-00332-5